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3-DIMENSIONAL PROBLEMS

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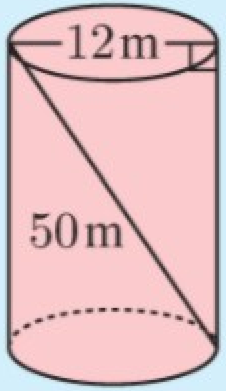
Pythagoras' theorem is often used to find lengths in **3-dimensional** problems.

Example 15

Self Tutor

A 50 m rope is attached inside an empty cylindrical wheat silo of diameter 12 m as shown. How high is the silo?



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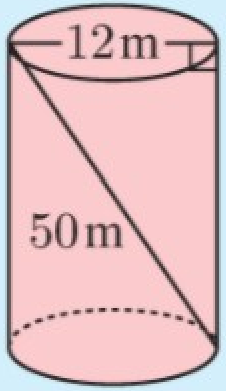

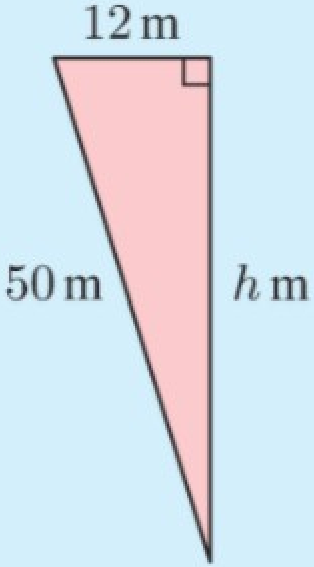

Let the height be h m.

$$\therefore \quad h^2 + 12^2 = 50^2 \qquad \{\text{Pythagoras}\}$$

$$\therefore \quad h^2 + 144 = 2500$$

$$\therefore \quad h^2 = 2356$$

$$\therefore \quad h \approx 48.5 \qquad \{\text{as } h > 0\}$$

The wheat silo is about 48.5 m high.

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Example 16 **Self Tutor**

A room is 6 m by 4 m, and has a height of 3 m. Find the distance from a corner point on the floor to the opposite corner point on the ceiling.

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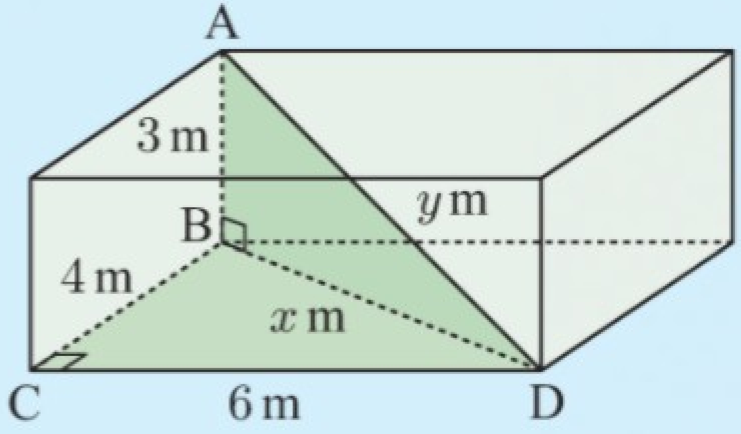
A room is 6 m by 4 m, and has a height of 3 m. Find the distance from a corner point on the floor to the opposite corner point on the ceiling.



The required distance is AD. We join [BD].

In $\triangle$BCD, $x^2 = 4^2 + 6^2$ {Pythagoras}

In $\triangle$ABD, $y^2 = x^2 + 3^2$ {Pythagoras}

$\therefore\ y^2 = 4^2 + 6^2 + 3^2$

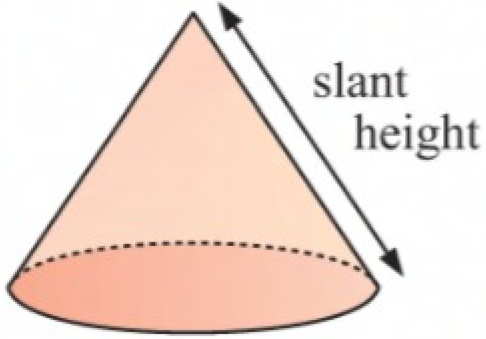
$\therefore\ y^2 = 61$

$\therefore\ y \approx 7.81$ {as $y > 0$}

$\therefore$ the distance is about 7.81 m.

EXERCISE 5F

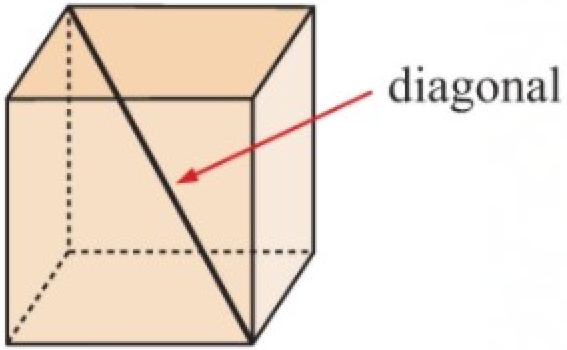
1 A cone has a slant height of 17 cm, and a base radius of 8 cm. How high is the cone?



2 Find the length of the longest nail that could fit entirely within a cylindrical can with radius 3 cm and height 8 cm.

3 A 20 cm nail just fits inside a cylindrical can. Three identical spherical balls need to fit entirely within the can. What is the maximum radius each ball could have?

4 A cube has sides of length 3 cm. Find the length of a diagonal of the cube.



5 A room is 5 m by 3 m, and has a height of 3.5 m. Find the distance from a corner point on the floor to the opposite corner of the ceiling.

6 A rectangular box has internal dimensions 2 cm by 3 cm by 2 cm. Find the length of the longest toothpick that can be placed within the box.

7 Can an 8.5 m long piece of timber be stored in a rectangular shed which is 6 m by 5 m by 2 m high?

Example 17 **Self Tutor**

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Determine the length of the slant edges.

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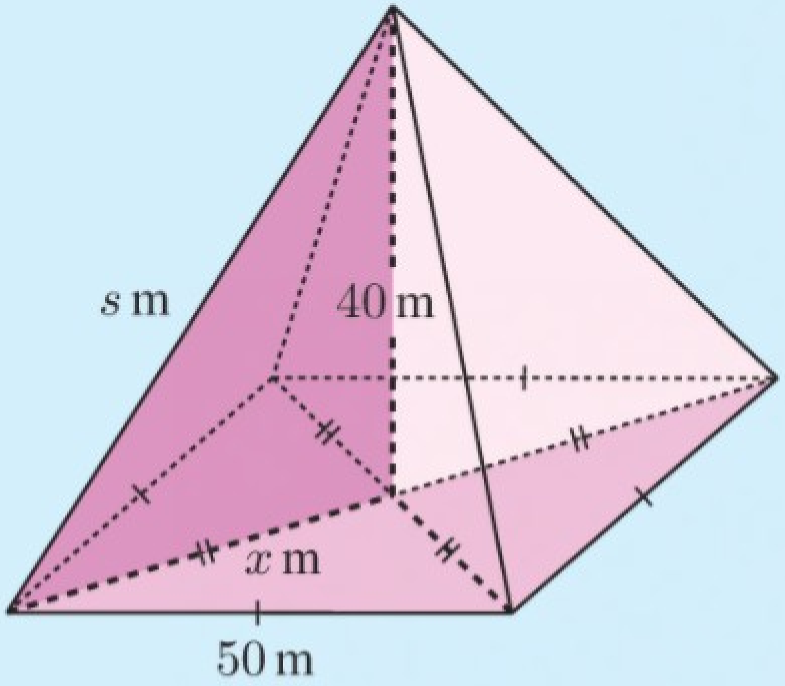
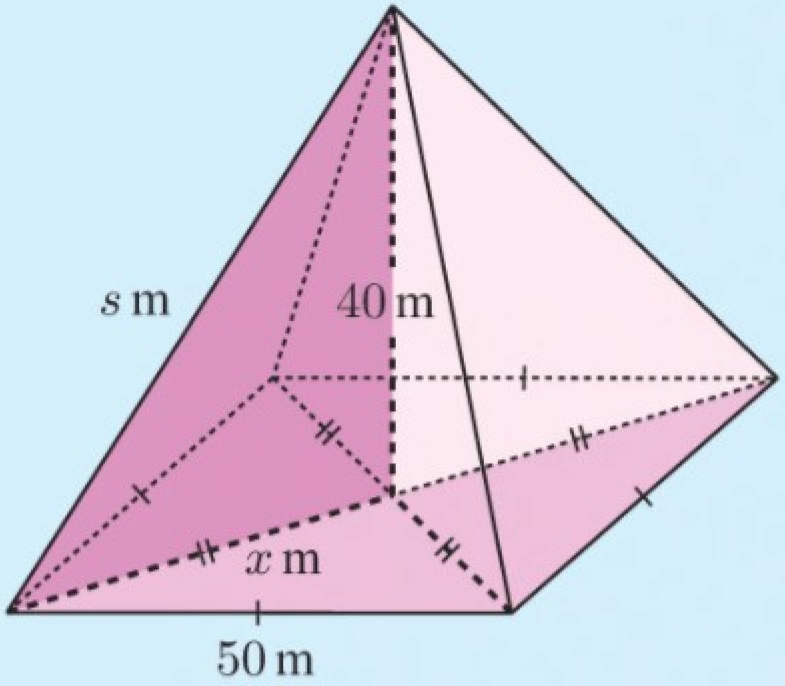


Example 17

Self Tutor

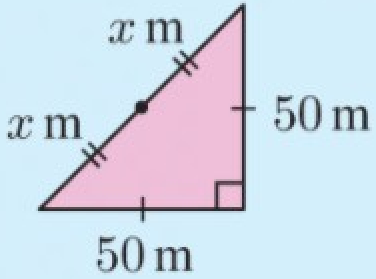
A pyramid of height 40 m has a square base with edges of length 50 m. Determine the length of the slant edges.



Let a slant edge have length s m.

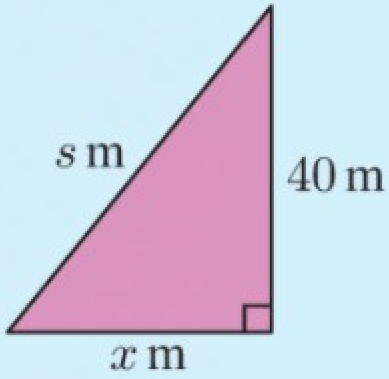
Let half a diagonal have length x m.

Using



$$\begin{aligned} (2x)^2 &= 50^2 + 50^2 \quad \{\text{Pythagoras}\} \\ \therefore\ 4x^2 &= 5000 \\ \therefore\ x^2 &= 1250 \end{aligned}$$

Using



$$\begin{aligned} s^2 &= x^2 + 40^2 \quad \{\text{Pythagoras}\} \\ \therefore\ s^2 &= 1250 + 1600 \\ \therefore\ s^2 &= 2850 \\ \therefore\ s &\approx 53.4 \quad \{\text{as } s > 0\} \end{aligned}$$

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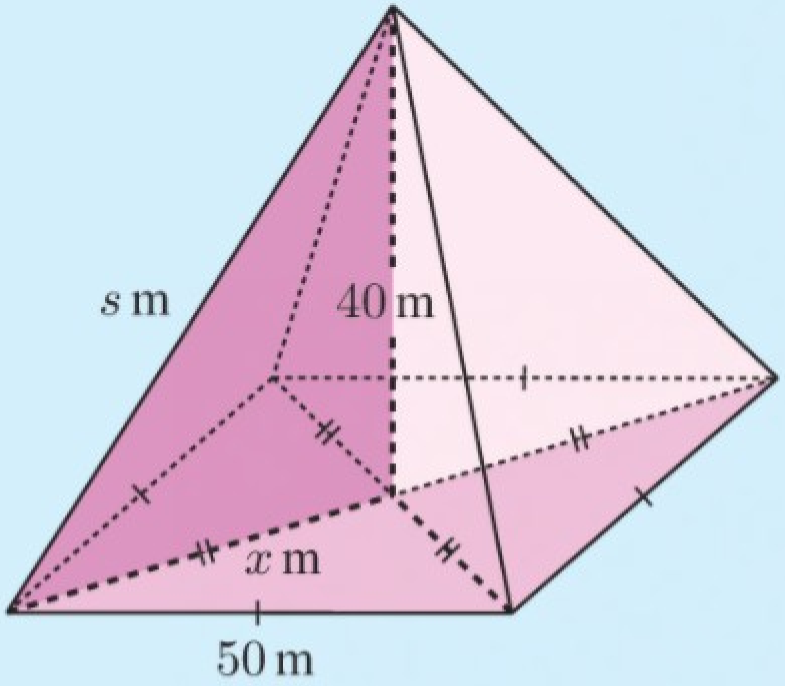
Determine the length of the slant edges.



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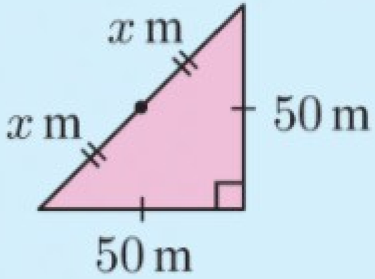

$$(2x)^2 = 50^2 + 50^2 \quad \{\text{Pythagoras}\}$$
$$\therefore\ 4x^2 = 5000$$
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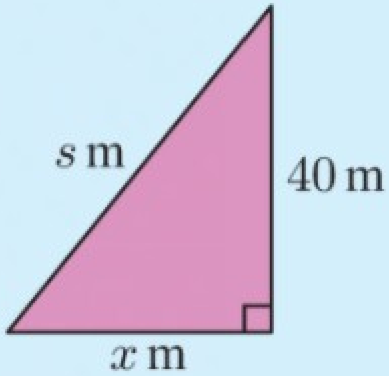

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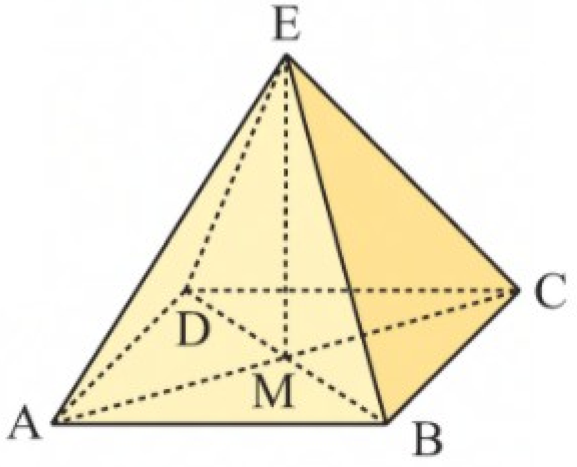
$$\therefore\ s^2 = 1250 + 1600$$
$$\therefore\ s^2 = 2850$$
$$\therefore\ s \approx 53.4 \quad \{\text{as } s > 0\}$$

Each slant edge is about 53.4 m long.

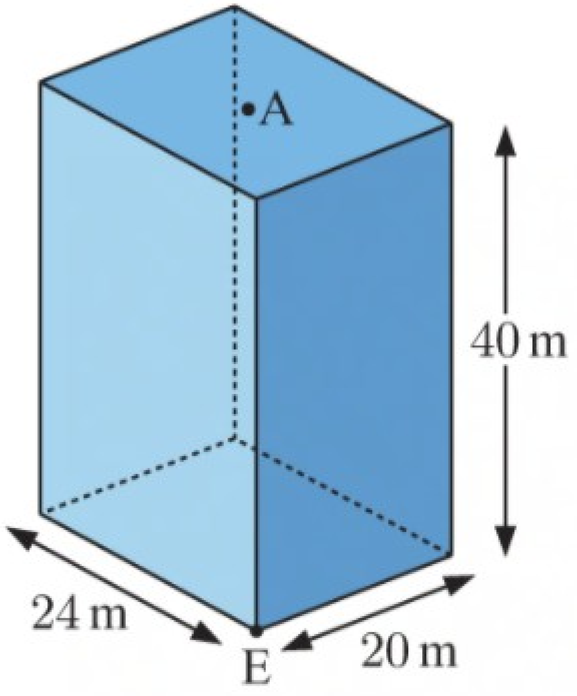
8 An Egyptian Pharaoh wishes to build a square-based pyramid with all edges of length 100 m. Its apex will be directly above the centre of its base. How high, to the nearest metre, will the pyramid reach above the desert sands?



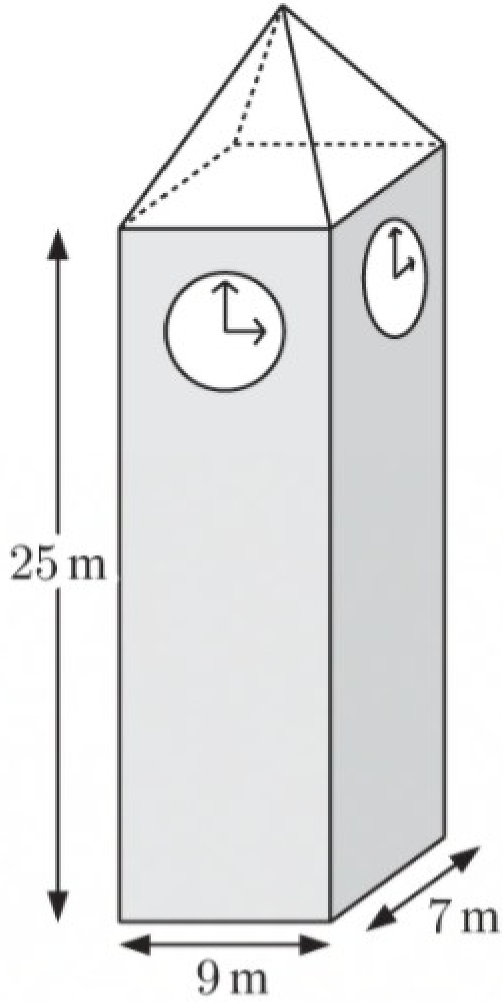
9 A symmetrical square-based pyramid has height 10 cm and slant edges of length 15 cm. Find the dimensions of its square base.

10 In the office building alongside, the entrance is at E. A radio antenna at A, the centre of the roof, receives security signals from the door E.

Find the direct distance between the entrance and the antenna.

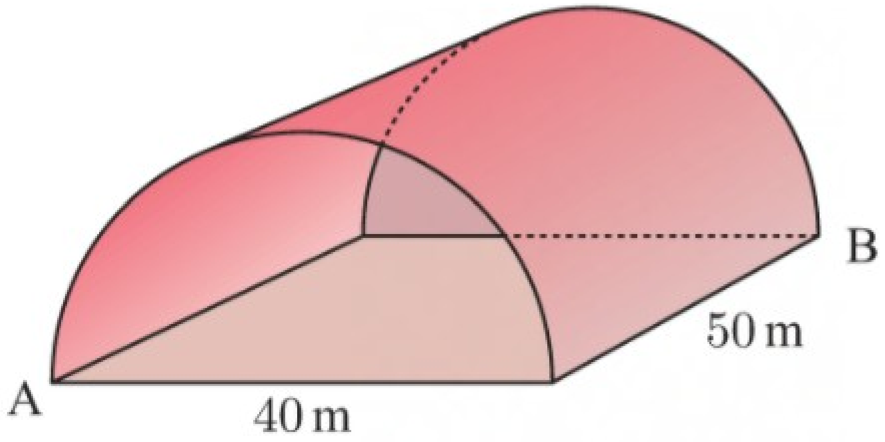


11



A clock tower has the dimensions shown. The slant edges of the pyramid are 7.5 m long. Find the height of the tower, to the nearest cm.

12 An aircraft hangar is semi-cylindrical with diameter 40 m and length 50 m. A helicopter places a cable across the top of the hangar, and one end is pinned to the corner at A. The cable is then pulled tight and pinned at the opposite corner B. Determine the length of the cable, to the nearest cm.



EXERCISE 5F

1 15 cm

2 10 cm

3 $\sqrt{10}$ cm ≈ 3.16 cm

4 $3\sqrt{3}$ cm ≈ 5.20 cm

5 ≈ 6.80 m

6 ≈ 4.12 cm

7 No, the maximum length would be about 8.06 m.

8 $\text{height} = \frac{100}{\sqrt{2}} \approx 71\text{ m}$

9 $5\sqrt{10}$ cm $\times$ $5\sqrt{10}$ cm ≈ 15.8 cm $\times$ 15.8 cm

10 ≈ 42.9 m

11 ≈ 29.87 m

12 $L = \sqrt{50^2 + (20\pi)^2}\text{ m} \approx 80.30\text{ m}$